Appendix 1 to CarlinoE, Sigauda M, Pollo A, et al. Nonlinear analysis of electroencephalogram at rest and during cognitive tasks in patients with schizophrenia. *J Psychiatry Neurosci* 2012.

DOI: 10.1503/jpn.110030

One-dimensional electroencephalography (EEG) data were transformed into multidimensional phase space. The phase space concept is crucial for nonlinear dynamics analysis. In a hypothetical system governed by *n* variables, the phase space is *n*-dimensional. Each state of the system corresponds to a point in phase space, with *n* coordinates that are the values assumed by the governing variables for this specific state. If the system is observed for a period of time, the sequence of the point in phase space converges into a subspace, called system's attractor. The technique of delay coordinates is carried out to reconstruct the attractor's trajectory. To unfold the projection back to a multivariate state space, a representation of the original system, the delay coordinates $y(t) = [x_j(t), x_j(t + T), \ldots x_j(t + [d - 1]\, T)]$ from a single time series x_j and the embedding procedure were performed. In this formula, y(t) is a point of the trajectory in the phase space at time t, x (t is iT) are the coordinates in the phase space corresponding to the time-delayed values of the time series, T is the time delay between the points of the time series considered, and *d* is the embedding dimension.[1,2] The choice of the optimal time delay T and embedding dimension *d* are important to the success of reconstruction with finite data. For the time delay, the first local minimum of the average mutual information between the set of measurements v(t) and v(t + T) was used in the present study. For the embedding dimension *d* the minimum (optimal) embedding was computed in the reconstruction procedure.[3] The attractor's reconstruction is necessary for the analysis of mathematical quantities characterizing the attractor itself. One important metric property of the attractor is the correlation dimension (D2), which estimates the degrees of freedom, such as the number of independent variables necessary to describe the dynamic of the original system. The larger the D2 values of the attractor, the more complicated the behavior of the nonlinear system. The D2 is thus a measure of the complexity of the process being investigated, and it characterizes the distribution of points in phase space.[4] The Gassberger–Procaccia algorithm[5] was computed to evaluate D2 values of the attractors from EEG data. With this algorithm, D2 is derived by determining the relative number of pairs of points in the phase-space set separated by a distance less than *r*. It is computed using the following equation: $\lim_{r\to 0} \lim_{N\to\infty} \frac{\log C(r,\ N)}{\log r}$

In this formula, the distance *r* is a radius around each reference point x_i; *N* is the number of data points in the phase space; *C* is the correlation integral, defined by $\frac{1}{N^2} \sum_{\substack{i,j=1 \\ i\neq j}}^{N} \theta\left(r - \left|\vec{x_i} - \vec{x_j}\right|\right.$, where x_i and x_j are the points of the trajectory in the phase space, and θ is the Heaviside function, defined as 0 if $x < 0$, and 1 if $x \geq 0$.

References

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